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ABSTRACT

These units were developed during the summer of 1987 as one component of a project on "Integrating Problem Solving Into Mathematics Teaching." Each unit has the objective of developing students' thinking abilities in mathematics. These units have been revised subsequent to field try-outs prior to dissemination. The summaries of 56 different teaching units, organized by levels (K-3, 4-6, 7-9, 10-12), are provided in this document. Each includes a title, a brief description, and the author's name. A list of author names and addresses is appended. (CW)

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Integrating Problem Solving
Into
Mathematics Teaching

SUMMARIES OF PROBLEM SOLVING TEACHING UNITS

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PREFACE

The summaries of Teaching Units which follow include the title, name of the teacher who developed the Unit, and a brief description of the Unit's content. The Units are organized by grade levels (K-3, 4-6, 7-9, 10-12) and are arranged alphabetically by author name within each group.

The Teaching Units were developed during the summer 1987 as one component of a project funded by the National Science Foundation on "Integrating Problem Solving Into Mathematics Teaching." Each Unit has the objective of developing students' thinking abilities in mathematics.

Teachers developed their Units during the on-campus project in the Summer, 1987 and then "tried them out" in their respective classes in Fall, 1987. Subsequently, teachers revised their Units and each Unit was again used by a different teacher in the project, at the same grade level, in Spring, 1988. Afterwards, each Unit was again revised by the Unit's author and the "second tryout" teacher, working together. Project staff later edited the Units for printing and dissemination.

The name and address of each Unit's author is given at the end of this Summaries of Problem Solving Teaching Units.

Anyone interested in having any of the Teaching Units summarized in the pages which follow may obtain them free by writing directly to the authors at the addresses given on pages 20-22.

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SUMMARIES OF TEACHING UNITS FOR GRADES K-3

Kindergarten Non-Number Activities

Julie Augustin

This unit is designed to guide kindergarteners in exploring non-number activities through the use of buttons. Through a variety of interactions, the students have an opportunity to sharpen their discriminating skills. Students learn to characterize objects by their attributes and relationships between objects or sets and also learn to sort according to chosen criteria. They determine whether two objects or sets are the same or different, according to a specific attribute or relationship. The students are given practice ordering and equalizing objects according to an attribute or relationship.

Relationships of Geometry

Steve Belcher

In this unit, students learn to classify and categorize according to shape, color, and size. They are introduced to elementary set ideas including logical connectives (and, or, both, not) and set membership. The activities in this unit use attribute blocks to teach children to organize their thoughts by grouping and sorting and to think logically. The unit is appropriate for a grade 3 classroom.

Which Way Should I Go? Finding the Shortest Path

Barbara Bierman

This unit for first through third grades is aimed at leading the student to discover that there is normally more than one way to get to a destination and that several ways may be correct. Each student is exposed to graphs, geoboards, and calculators. Each student also experiences transferring designs to paper and working with maps including planning a trip on a primary map (or a real map if the Day 5 extension is used). The unit is organized for four 45-minute periods (five days if Day 5 extension is used). The unit includes teacher-directed activities as well as group and team projects.

Problem Solving With Money

Marie Davis

This unit is intended to teach the relative values of money using the circle, triangle, square, and hexagon as manipulatives. It is organized into seven sessions. Prerequisite skills include earlier introduction to money, the ability to add and subtract, and some experience in place value in base 10 and in base 5.

Spatial Visualization Unit

Mary M. Davis

This unit is intended for students in grades 3-6. The three activities help students visualize three-dimensional solids from two-dimensional drawings. The activities move back and forth from concrete experiences (three-dimensional building blocks--in this case cubes) to abstraction (two-dimensional drawings). Students are encouraged to stack cubes to match drawings and ultimately count the number of cubes in drawings without the use of the concrete building blocks.

Beginning Logo Unit

Barbara A. Duncan

This unit includes eight 1-hour lessons on using the computer program LOGO (or turtle math) for beginners. Since LOGO is filled with problem solving, students will discover, follow directions, create, use logic, use trial and error, and have loads of fun on the computer. The unit includes directions for using both a computer and LOGO, worksheets for students, transparencies and computer printouts. Students build strong concepts in geometry by drawing shapes using both lengths of sides and measures of angles. Skills from the previous lesson are constantly reinforced. Although the unit was designed for third grade students, it may be used for all elementary grades.

Triangles

Cheryl Martin

This geometry unit is designed for third grade and involves a thorough investigation of the triangle. The unit is organized into a pretest, seven lessons, and a posttest. Lesson 1 should take about one hour of class time. Lessons 2 through 7 should take about two hours each. The seven lessons cover the following topics: Recognizing a Triangle, Dividing a Rectangle, Drawing a Triangle, Polygons From Origami Triangles, Summing the Angles of a Triangle to 180 Degrees, Dividing Polygons into Component Triangles, and Computing the Number of Degrees in a Polygon. The unit may also be used in higher grades.

Introduction to Problem Solving

Dolores McNabb

This problem solving unit is designed for third graders and is organized for five days of instruction early in the school year. Students are encouraged to learn that there are many ways to find a solution to a given problem. The unit emphasizes various heuristics, including using a table, finding a pattern, trying and checking, eliminating possibilities, using logical thinking, using physical models, working backwards, listing all possibilities, solving a simpler problem, and solving a similar problem. Individual, group, and team activities are used in the unit.

An Introduction to Geometry

Beth Patton

This unit is designed for third grade students and is organized for ten days of instruction. It focuses on several aspects of geometry including shape, direction, patterns, sets, area, and direction. Hands on activities are strongly emphasized using manipulatives such as attribute blocks, tangrams, geoboards, and an Apple Logo computer program. While experiencing this unit on geometry, students are also introduced to some heuristics of problem solving.

Metric Line Measurement

Carol Ritchey

This third grade unit develops the concept of a unit for linear measurement. Students begin by choosing and cutting units of any size. Then they measure various lengths with these units. After the need for a standard unit is established, lesson 2 has students estimating and measuring the lengths of small objects using metric rulers. Lesson 3 leads students to discover that a meter equals 100 centimeters. The children make meter sticks and measure the lengths of larger objects. In lesson 4 students measure around objects (perimeter), and in lesson 5 they work with more abstract measurements using maps.

Numbers for Kindergarten

Lynette Schaefer

This kindergarten unit combines many skills, including listening and direction skills, while preparing students to eventually use Logo. Students use a grid and learn to determine how many steps from one point to another. They use the grid to compare lengths of paths and learn to determine the shortest and longest paths and distinguish between them. Students experience finding different ways to go from one point to another by using ten steps. They use direction words to have the Logo turtle form boxes on the grid, learn to make and read a bar graph, and count the square units inside different rectangular shapes.

Fun for the First Five Days

Imajean Snell

This unit is designed for use at the beginning of the year, ideally the first week of school, for first and second grade students. It gives students a preview of manipulatives that will be used in the math classroom. The unit contains a sample of activities using Attribute Blocks, Geoboards, Calculators and LOGO (on the computer). Each activity uses a different manipulative. Alternate ideas are given in case Attribute Blocks or Geoboards are unavailable; however, a calculator and LOGO program are needed for some activities. The unit also contains an activity on data collection using watermelons. This week of fun will encourage a positive attitude toward mathematics and problem solving.

Using Measurement to Solve Mathematical Problems

Rebecca L. Stewart

This four-day unit provides second grade students the opportunity to collect measurement data and organize it in graph and chart form. Students are challenged with problem solving activities requiring the use of formal and informal units of measurement. Graphs and charts enable students to organize, compare, and interpret data.

Geometry the Geoboard Way

Deborah Yates

This third grade unit gives students an opportunity to work with geometric concepts at a concrete level. Students gain experience with two-dimensional shapes and measure perimeters and areas on geoboards and centimeter dot paper. Students learn to identify and construct similar and congruent figures on the geoboard and centimeter dot paper. Tangrams are used to form various geometric patterns.

SUMMARIES OF TEACHING UNITS FOR GRADES 4-6

Teaching Statistics in the Sixth Grade

Cynthia Bremer

This unit for sixth grade students explores the use of the stem-and-leaf plot as a tool for organizing, comparing and analyzing data. Students are given data on M and M's, ice cream, sports, and motorcross biking. They are encouraged to become familiar with the data before organizing or analyzing it. The unit encourages skills in reading, writing, verbalizing, and problem solving. An introduction to stem-and-leaf plotting is provided within the activities.

An Introduction to Problem Solving

Kelly Buntin

This third grade unit on basic problem solving emphasizes the "four-step" approach by George Polya:

1. Understand the problem
2. Devise a plan or strategy to solve it
3. Solve the problem--does it make sense?
4. Can you find and solve extensions to the problem?

The unit is planned for four periods, approximately one day each week. It includes a letter to parents explaining the unit, an outline for an open house to get parents involved in the excitement of problem solving, class-problems (ready for xerox), teacher's guide for each set of problems, and more problems to take home. The unit also incluides posters for a bulletin board, awards and a game. The unit may also be used in higher grades.

An Introduction to Problem Solving Strategies

Theresa Douglas

This unit is designed to instruct parents and pupils on how to use five specific problem solving strategies: trial and error, find a pattern, draw a picture, make a chart, and work backwards. It would be most effective as a beginning of the year activity. The unit includes specific teaching instructions, practice, extension, and a parent problem for each strategy. Bulletin board graphics and extra credit problems are also provided.

Problem Solving Teaching Unit

Connie Frerker

This unit provides an introduction to problem solving for fourth grade students. The unit is based on the heuristics of George Polya with emphasis on process and thinking skills. The theme of the unit is to develop initial success. The problems and activities are designed to stimulate interest and generate positive feelings toward mathematics in general and problem solving in particular. The unit is organized in a booklet format for use at the beginning of a school year.

Magic Squares

Earnest Gardner

This unit uses the topic of magic squares to introduce fourth grade students to problem solving. A common method for finding a solution to a magic square is trial and error; but the unit leads students to look for patterns and combinations as they search for the solution. The ultimate goal of the unit is to encourage students to develop a generalized method or strategy for solving a magic square.

Criss-Cross Multiplication

Gilbert S. Gee

This unit presents a new and different approach to multiplication. If it is taught with an emphasis on problem solving, criss-cross multiplication can be a nice activity. Students are taught the criss-cross multiplication method in one or two class periods, and for the next few periods they discover by themselves how to apply the method to more difficult problems using larger numbers. The last periods are critical to reinforce understanding the new algorithm..

Matchstick Shapes

Diann Grace

This fourth grade unit is an introductory experience in problem solving. Students are exposed to George Polya's three characteristics of a problem:

1) Acceptance
2) Blockage
3) Exploration

The students are introduced to the four-step approach by Polya:

1) Read and understand the problem.
2) Devise a plan to solve the problem.
3) Solve the problem.
4) Check the problem and create new problems.

The unit uses the "Match Stick Problem" to get students involved in problem solving.

Find a Pattern

James Grace

This problem solving unit is directed at the intermediate/middle school levels and is organized for use in one week. Students are taught to use George Polya's four-step approach to solving a problem:

a) Read and understand the parts of the problem.
b) Devise a plan to solve the problem.
c) Solve the problem.
d) Check to see if there is a situation for which the solution will not work. Extend the solution to related problems.

The unit concentrates on the strategy of finding a pattern as it helps students develop higher level thinking skills.

The Logical Answer

Melinda G. Heins

This unit for grades 4-8 includes 30 logic puzzles with brief instructions for teaching students to solve them with the aid of a chart (matrix). Each puzzle is ready for duplication and answers are included for teachers. The unit can be taught over a one or two week period (1 period per day) or the puzzles can be given over several months (letting students work at their own pace).

Line Plots and Patterns

Marilyn McCallister

The major purpose of this unit is to develop confidence in problem solving skills while introducing middle grade students to the world of statistics and patterns. Emphasis is placed on process, rather than solely on correct answers. Students are introduced to simple data analysis, the construction of line plots, and to the concepts of mean, median, clusters, gaps, and outliers.

Problem Solving - An Introduction

Kay Reeder

This fifth grade unit provides an introductory experience in problem solving. In the first three lessons, students use basic concepts of set theory with shapes, whole numbers and area. They are encouraged to compare various properties and categorize while using attribute blocks, cards, and a geoboard (square grid). In lesson 4, students interpret and solve story problems by drawing pictures. In lesson 5, students collect data with shelled peanuts and graph the results on a line plot. This unit may also be appropriate for lower grade levels, having been used successfully at grades K-5.

Problem Solving Unit

Ben Wesselman

This unit is appropriate for middle and upper elementary school students. It is to be used over a period of four weeks. Each week includes a class problem, an independent problem, a group problem and several extra challenge problems. Students are expected to discuss and record strategies, procedures and solutions. Students, in fact, record this information on a worksheet outlining Polya's 4-step approach:

1. Understand the Problem
2. Devise a Plan (Solving Problem)
3. Problem Solution
4. Extensions

Completed worksheets are included as examples.

SUMMARIES OF TEACHING UNITS FOR GRADES 7-9

Developing Estimation Skills Through Problem Solving

Sharon Carney

This unit is designed for seventh grade students. They observe burning candles that are covered by glass jars of various sizes. Students combine math and science by experimenting, estimating, averaging, completing a chart, graphing observations, analyzing graphs, and making and testing predictions. The unit includes worksheets providing practice in estimating answers to computation problems and solving proportions and percentage problems.

Teaching Unit on Fractions

Charles Dreessen

This unit is designed as a review of fractions for middle school students. Addition and subtraction problems are given with an emphasis on problem solving. Although the computation is reviewed before each day's assignment, the problems are, for the most part, non-routine thinking problems. For example, students fill in missing blanks (circles) in equations, solve arithmagons, magic squares, puzzles, and word problems. The unit includes daily worksheets for students and notes with example problems for teachers. The unit also includes a page of suggestions for the teacher based on experience gained from teaching the unit.

Teaching Unit - Fractions

Roberta Hawthorne

This unit is designed for sixth grade students and is organized for six days of instruction. An introduction to problem solving is given and students solve several fraction problems using strategies of "guess and check" and "work backwards." Examples of problems include magic squares, two long "thinking problems" and routine word problems.

Patterns

Gae Irby

This unit leads students through a discovery of number patterns. Students work with adding consecutive integers, arithmetic and geometric progressions, and Pascal's Triangle. The problem solving approach in this unit gives students experience with the heuristic, or learned strategy, of looking for patterns.

Introduction to Problem Solving

Don Kittinger

This unit is designed for middle school, as a one-week problem solving unit. Students are required to solve 5 problems (and their extensions) using different strategies suggested by George Polya. They are also encouraged to write computer programs that model the problems. Several have been included for teacher reference. Problems in the unit may also be appropriate for higher grade levels.

Determining the Number of Objects in Various Arrays

Michael Koenig

This unit includes activities that are very good for building the concepts of area and volume. Students are given a number of objects that border a regular geometric shape (all sides equal). They determine the number of objects inside the figure and then extend the activity to three-dimensional objects. For all problems, students are encouraged to make generalizations. The unit may be appropriate for upper elementary as well as middle school students.

Problem Solving in Mathematics for the Middle Grades

Dolores Lane

This unit is designed for students in grades 6-8, but can also be used for general math students in high school or advanced students in elementary school. The unit encourages students to solve problems by:

Making a Drawing
Guessing and Testing
Eliminating Possibilities
Looking for Multiple Solutions

The first two days are devoted to problems that can be solved by making a drawing; the third and fourth are devoted to problems that can be solved by guessing and testing; and so on for 8 days. The problems have been carefully chosen to provide both challenge and success, encourage ownership in the problem solving process, and to produce creativity. A point system for evaluation is included.

Geometric Numbers

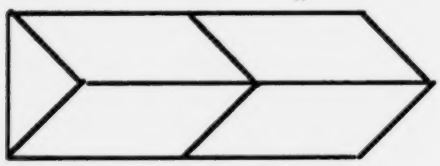
Judith Lecocq

This unit leads students through a discovery of triangular, square, cubic, and pentagonal numbers. Students gain valuable experience recognizing patterns and using algebraic expressions to generalize problems. The unit includes lists of suggested teacher questions and/or comments. It is designed for 4 or 5 class periods.

A Teaching Unit on Euler's Circuits

David Lewis

This unit is designed for students in grades 5-8, but may also be appropriate for students of various grades and with wide ranging abilities. Students explore an important topic in discrete math: the traceability of Euler graphs. Many graphs of the type below



are given and students decide whether or not they can be traced in such a way that every edge is covered exactly once, without lifting the pencil. Students are permitted to touch a vertex (point) as many times as needed. Several generalizations about the traceability of graphs are discovered. Students are also given interesting application problems.

Volume

Molly Jo McDaniel

This unit is designed for junior high science students. In lessons 1 and 2, students solve problems of the type: "Suppose a beaker has only 700 ml, 500 ml, and 300 ml markings. Could you measure exactly 400 ml?" Students are encouraged to experiment with beakers and water. Students not only gain lab experiences, but are placed in problem solving situations that require critical thinking. Problems increase in difficulty and extensions are given. In lesson 3, students experiment with ice and water to compare the volume of a solid with the volume of liquid. They are led, again through experimentation, to the fact that 1 cubic centimeter = 1 milliliter. In lesson 4, they use charts, lab work, patterns and discovery to extend this idea.

Number Theory

Ernestine Mitchell

This unit leads students through an exploration of factors and multiples. Written with a discovery and problem solving approach, it uses games and discovery exercises that encourage the student to look for and find patterns and generalizations.

Solving Word Problems

Marvin R. Pigg

This unit is appropriate for upper elementary school and middle/junior high school students. Word problems of five types are given: one-step problems using whole numbers, one step problems using fractions, two-question problems, two-step problems and percentage problems. George Polya's approaches to solving problems are given and teachers are given the opportunity to use their creativity in applying Polya's ideas to the problems.

Problem Solving in the Classroom

Jerry R. Scott

This unit is designed for ninth grade general math students. It includes a thorough and informative explanation of problem solving and helpful techniques for incorporating problem solving into the classroom. Problems are included that use the following strategies: exploring patterns with calculators, generalizing number patterns, trial and error, logic, and making a chart, table or graph. Problems are non-routine in nature and the unit encourages a process for solving rather than finding an answer. The unit is designed for five lessons. In the first four, specific strategies are demonstrated but not required for the sample problems. In lesson five, students independently choose appropriate methods for solving a set of problems.

Teaching Unit on Geometry

Ruby H. Smith

This unit on geometry can be used at the seventh grade level. It is intended to enrich and reinforce understanding of various geometrical concepts in a real life setting by using the students' natural environment.

Fewest Number of Tacks

Jill Winthrop

This unit is designed for middle school students and may also be appropriate for elementary students. Students spend 3 lessons on the problem of tacking paper on a bulletin board, using the fewest number of tacks. In lesson 1, they are instructed on George Polya's four step approach to solving a problem. In lessons 2 and 3, they use the approach to solve the tack problem. Often teachers give extensions to problems; however in this unit, the students are also encouraged to think of extensions and write new (related) problems. The tack problem uses strategies of drawing a picture, making a chart, looking for patterns and finding generalizations.

SUMMARIES OF TEACHING UNITS FOR GRADES 10-12

Venn Diagram Game

Rhonda Griffey Adkins

This unit requires students to determine the rule for classification of geometric shapes. It includes simple levels using only one criteria for inclusion or exclusion and extends to more complex levels using three intersecting criteria. While playing the game, students experience problem solving activities such as using guess and check, the process of elimination, and Venn Diagrams as problem solving tools. Students also experience activities that are prerequisite to an understanding of sets and set theory.

An Introduction to Trigonometry

Deana L. Brashear

This unit teaches trigonometric concepts using a problem solving approach. Students work outdoors to solve three problems:

1. Find the height of a flagpole.
2. Calculate the altitude of a kite.
3. Stake out a right triangle.

Students are equipped with a string, kite, meter stick, tape measure and metal stakes. The problems can be solved by making small measurements, finding ratios, setting up proportions, and applying the Pythagorean theorem and the concept of similar triangles. Students work in groups and are given guidelines for each group to follow. The unit provides alternate questions (if inclement weather precludes going outdoors). The unit is appropriate for students who have had high school geometry.

Introducing Problem Solving to Pre-Calculus Students

SuEllen Brauer

This unit is designed for pre-calculus students or students who have had four semesters of algebra and two semesters of geometry. The unit gives an outline of geometry concepts and theorems. It contains problems with geometric constructions, graphing, puzzles and other challenging thinking problems. Students work in groups and solve problems using varied and creative strategies.

Handshakes???

Mary Lou Hickam

This unit is designed for Pre-Algebra and Algebra I students who are not accelerated in mathematics. Six problems are given with worksheets for students and notes for the teacher. These non-routine thinking problems lend themselves to creating a chart, looking for patterns and making generalizations. They also encourage counting skills that are necessary in probability and combinatorics, in that students need to consider all possible combinations to solve many of the problems. Most of the problems could be modified for use in a probability context, if that is desired.

Teaching Unit on Problem Solving

Halsie E. King

This unit is to be implemented at the beginning of the year for high school students. It may be modified for lower grade levels. The unit is well organized, with daily student worksheets and teacher notes. Several charts and pictures are designed to illustrate solutions and provide tools for solving various problems. The problems give practice in using logic (cryptograms and magic circles), analyzing patterns (locker and calculator problems), developing organized methods of counting (random path problem), and working backwards to arrive at a solution (word problems).

An Introduction to Logic

Becky Mandrell

This unit is designed for students in tenth grade geometry. It may also be used for younger students and more advanced students. The unit encourages group work with logic problems. It is intended as a five-lesson experience; however, some problems are difficult to solve and students and teacher might be interested in extending the allotted time. One of the highlights of this unit is the fact that several of the problems contain more than one possible outcome (answer). Students can be encouraged to analyze these problems in depth.

A Teaching Module for Problem Solving and Mixture Problems

Helen Meseke

This unit introduces students to mixture problems commonly found in high school algebra and chemistry. The unit treats many examples of the same type of problem, and is designed for six class periods using a problem solving approach.

Four Strategies for Solving Problems

Christy Peter

This unit investigates four problem solving heuristics: drawing pictures, simplifying, guessing and checking, and working backwards. The unit is designed for six lessons in a beginning Algebra I class. The model for the unit is as follows:

1. Introduce the day's strategy.
2. Work an example, designed to incorporate Polya's four-step approach.
3. Students work problems in groups, using the day's strategy.
4. Unfinished problems can be used as homework.

The first and last lessons are designed as an introduction to the unit and an evaluation, respectively. A nice set of problems has been provided and the time allotment for the unit could be doubled, spending two days on each strategy and two days on evaluation (group presentations).

Newspaper Numbers

Gary Risenhoover

This unit uses articles, ads, tables and graphs from the newspaper to make math meaningful and interesting. Some of the teaching ideas can be followed exactly and some are given to encourage teacher creativity. To use the unit effectively, a current newspaper or shoppers guide (many copies) should be provided once a week for six weeks. The unit is intended for eighth through tenth grade general math students and may also be appropriate for other grades. In the unit, students are exposed to consumer math (making the best buys), analyzing a weather graph, and interpreting data from a diet ad and a school attendance article. Students also play the stock market.

Problem Solving With Order of Operations

Kitty Ann Scarlett

This unit is based on the following problem:

Find the integers 1 to 15, inclusive, that satisfy

$$(a + b) \times c = (a \times b) + c$$

In lesson 1, students examine four solutions and use them to find others. Of the 47 possible solutions, one pilot group of students found 43 in two days using the Fundamental Principle of Counting to find the number of possible permutations of the integers 1 to 3, then 1 to 4, and so on until they found a pattern and generalized a rule. After students discover that there are 3375 possible permutations, the teacher introduces the idea of using a microcomputer to find the last four. Students copy a program, print out solutions and immediately see the value of the computer as a useful tool. Each time a new strategy is discovered, students are encouraged to record it in their notebook. The unit may also be used with Pre-Algebra students with very good results.

Problem Solving Teaching Unit

John L. Schmitt

This unit is designed to introduce students to problem solving. It includes a good variety of non-traditional problems along with an introduction to Polya's 4-step plan for solving problems. The unit is divided into six lessons. Each lesson can be completed in one or two class periods.

Solving Maximum and Minimum Problems Without Calculus

Ann Schnurbusch

This unit approaches maxima and minima problems with a strong focus on problem solving. Students solve up to 25 interesting and challenging problems related to money (maximizing profits and minimizing costs), number puzzles, area, perimeter, volume and surface area. The unit is designed for Pre-calculus/Trig or Algebra II students and contains activities for five lessons. The unit introduces Heron's formula for the area of a triangle and the inequality associated with the arithmetic and geometric means. Students approach problems using several methods including use of a table to chart values, observation of maximum and minimum points associated with a quadratic function (parabola), and concepts of the arithmetic and geometric means. Students are encouraged to work in groups, use calculators and solve problems by methods of their choice. The unit includes teacher pages with objectives, strategies and solutions, and pages distributed for student use.

Encouraging Critical Thinking in Mathematics

Joe Wendling

This unit provides a good introduction to problem solving in five class periods. It contains many non-traditional problems (including some in geometry and logic) which require critical thinking. A pretest and a posttest are provided.

Planting Roses
(a non-routine problem)

Jane Woodside

This unit uses the theme of "rose bushes" to increase creativity and build spatial visualization skills. The first problem is to arrange 5 rose bushes in 2 rows, with 3 bushes in each row. By changing the designated number of rose bushes, rows, and bushes in each row, students experiment, draw pictures and explore patterns to find different solutions. An important characteristic of the unit is that it may be used for grades 3-12. The author encourages lower/elementary students to approach it with hands-on experiences, middle/junior high students to explore properties of geometric figures and number patterns, and senior high students to extend the activity to Euler paths, logic and analysis, finite differences and computer programming.

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